

Chapter-11Conic SectionsEx-11.1

$$1. (x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-2)^2 = 2^2$$

$$x^2 + y^2 + 4 - 4y = 4$$

$$x^2 + y^2 - 4y = 0$$

$$2. (x-h)^2 + (y-k)^2 = r^2$$

$$(x+2)^2 + (y-3)^2 = 4^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

$$3. (x-h)^2 + (y-k)^2 = r^2$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \frac{1}{12^2}$$

$$4. (x-h)^2 + (y-k)^2 = r^2$$

$$(x-1)^2 + (y-1)^2 = \sqrt{2}^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y + 2 = 0$$

$$x^2 + y^2 - 2x - 2y = 0$$

$$\frac{x^2+1}{4} - x + \frac{y^2+1}{16} - \frac{y}{2} = \frac{1}{144}$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 + 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

$$5. (x-h)^2 + (y-k)^2 = r^2$$

$$(x+a)^2 + (y+b)^2 = (\sqrt{a^2+b^2})^2$$

$$x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 + b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

$$6. (x+5)^2 + (y-3)^2 = 36$$

$$(x-(-5))^2 + (y-3)^2 = 6^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$h = -5 \quad k = 3 \quad r = 6$$

$$7. x^2 - y^2 - 4x - 8y - 45 = 0$$

$$(x^2 - 4x)(y^2 - 8y) = 45$$

$$x^2 - 4x + (2)^2 = (2)^2 + y^2 - 8y + (4)^2 - (4)^2 = 45 = 0$$

$$(x-2)^2 + (y-4)^2 - 4 - 16 - 45 = 0$$

$$(x-2)^2 + (y-4)^2 - 65 = 0$$

$$(x-2)^2 + (y-4)^2 = 65$$

On comparing

$$h = 2 \quad k = 4 \quad r^2 = 65$$

Centre $(h, k) = (2, 4)$
radius $= \sqrt{65}$

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9. $2x^2 + 2y^2 - x = 0$

Divided by 2

$$x^2 + y^2 - \frac{x}{2} = 0$$

$$x^2 - \left[\frac{1}{2}\right]^2 + \left[\frac{1}{4}\right]^2 - \left[\frac{1}{4}\right]^2 + y^2 = 0$$

$$\left[x - \frac{1}{4}\right]^2 + (y - 0)^2 = \left[\frac{1}{4}\right]^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$h = \frac{1}{4}$ $k = 0$ $r = \frac{1}{4}$ Centre $\left[\frac{1}{4}, 0\right]$ $r = \frac{1}{4}$

10. Let eqⁿ of circle is $(x-h)^2 + (y-k)^2 = r^2$ — (1)

(1) passes through (4, 1)

$$(4-h)^2 + (1-k)^2 = r^2 \text{ — (2)}$$

passes through (6, 5)

$$(6-h)^2 + (5-k)^2 = r^2 \text{ — (3)}$$

Centre (h, k) is on line $4x + y = 16$

$$4h + k = 16$$

$$k = 16 - 4h \text{ — (4)}$$

∴ subtract (3) - (2)

$$(6-h)^2 + (5-k)^2 = r^2$$

$$(4-h)^2 + (1-k)^2 = r^2$$

$$(6-h+4-h)(6-h-4+h) + (5-k+1-k)(5-k-1+k) = 0$$

$$(10-2h) \times 2 + (16-2k) \times 4 = 0$$

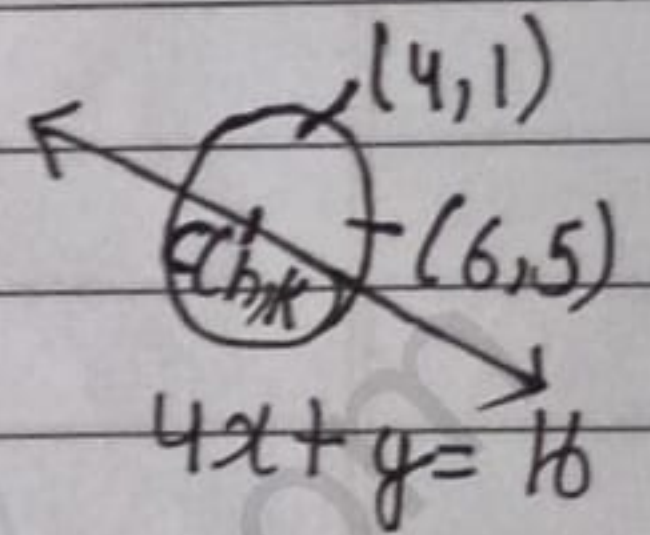
$$(5-h) \times 4 + (16-2k) \times 4 = 0$$

$$(5-h) \times 6 - 2 \times (16-4h) = 0$$

$$5-h+6-32+8h = 0$$

$$7h = 21 \quad h = 3$$

$$4k = 16 - 4 \times 3 \Rightarrow 16 - 12 = 4$$



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$$(4-3)^2 + (1-4)^2 = r^2$$

$$1+9 = r^2 \quad r = \sqrt{10}$$

$$(x-3)^2 + (y-4)^2 = 10$$

11. Let eqⁿ of circle is $(x-h)^2 + (y-k)^2 = r^2$ - (1)

passes through (2,3)

$$(2-h)^2 + (3-k)^2 = r^2 \quad \text{--- (2)}$$

passes through (-1,1)

$$(-1-h)^2 + (1-k)^2 = r^2$$

$$(1+h)^2 + (1-k)^2 = r^2 \quad \text{--- (3)}$$

Centre (h,k) lie on $x-3y-11=0$

$$h-3k-11=0 \quad \text{--- (4)}$$

$$\text{(2) - (3)}$$

$$(2-h)^2 + (3-k)^2 = r^2$$

$$(1+h)^2 + (1-k)^2 = r^2$$

$$(2-h+1+h)(2-h-1-h) + (3-k+1-k)(3-k-1-k) = 0$$

$$3(1-2h) + (4-2k)2 = 0$$

$$3-6h+8-4k=0$$

$$6h+4k=11 \quad \text{--- (5)}$$

$$4 \Rightarrow h = 3k + 11 \text{ in (5)}$$

$$6(3k+11) + 4k = 11$$

$$18k + 66 + 4k = 11$$

$$22k = 11 - 66$$

$$22k = -55$$

$$k = -\frac{5}{2}$$

$$\text{(4) } h = 3 \times -\frac{5}{2} + 11 = \frac{-15+22}{2} = \frac{7}{2} \quad \boxed{h = \frac{7}{2}}$$

$$\text{(3) } \left[1 + \frac{7}{2}\right]^2 + \left[1 + \frac{5}{2}\right]^2 = r^2$$

$$\left[\frac{9}{2}\right]^2 + \left[\frac{7}{2}\right]^2 = r^2$$

$$\frac{81}{4} + \frac{49}{4} = r^2 \quad r^2 = \frac{130}{4}$$

$$\text{Eq}^n \text{ of circle} = \left[x - \frac{7}{2} \right]^2 + \left[y + \frac{5}{2} \right]^2 = \frac{130}{4}$$

$$x^2 + \frac{49}{4} - 7x + y^2 + \frac{25}{4} + 5y = \frac{130}{4}$$

Multiply by 4

$$4x^2 + 49 - 28x + 4y^2 + 25 + 20y = 130$$

$$4x^2 + 4y^2 - 28x + 20y = 56$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

12.

$r = 5$ unit

Let centre $(h, 0)$

$$\text{Eq}^n \text{ of circle} = (x-h)^2 + (y-0)^2 = 5^2$$

$$(x-h)^2 + y^2 = 25 \quad \text{--- (1)}$$

Passes through $(2, 3)$

$$(2-h)^2 + (3)^2 = 25$$

$$(2-h)^2 = 25 - 9 = 16$$

$$(2-h) = \pm 4$$

$$2-h=4 \Rightarrow 2-4=h \Rightarrow h=-2$$

$$2-h=-4 \Rightarrow 2+4=h \Rightarrow h=6$$

$$(x+2)^2 + y^2 = 25$$

$$x^2 + 4 + 4x + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

$$x^2 + y^2 + 4x - 21 = 0$$

$$(x-6)^2 + y^2 - 25 = 0$$

$$x^2 + 36 - 12x + y^2 - 25 = 0$$

$$x^2 + y^2 - 12x + 11 = 0$$

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Centre $(2, 2)$ point $(4, 5)$

eqⁿ of circle :- $(x-2)^2 + (y-2)^2 = r^2$

passes through $(4, 5)$

$$(4-2)^2 + (5-2)^2 = r^2$$

$$4 + 9 = r^2$$

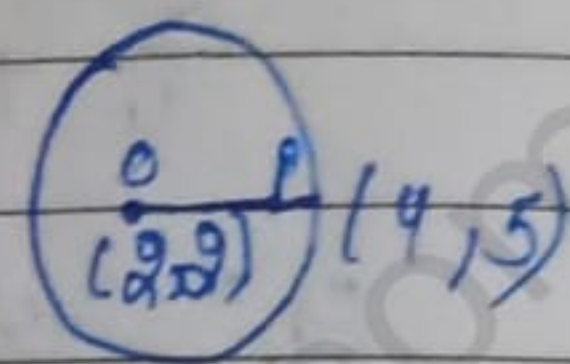
$$r^2 = 13$$

$$(x-2)^2 + (y-2)^2 = 13$$

$$x^2 + 2^2 - 4x + y^2 + 2^2 - 4y(-2) = 13$$

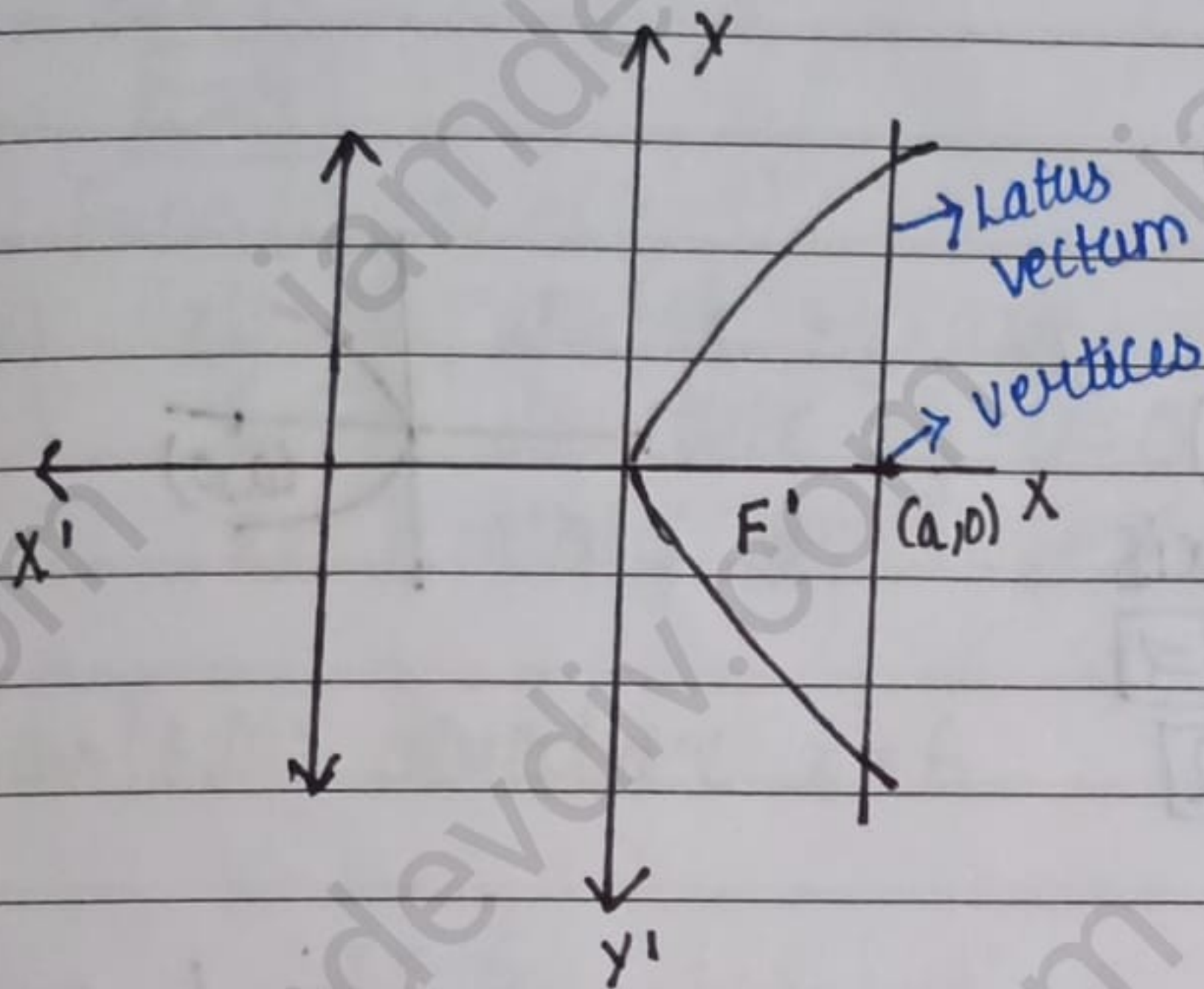
$$x^2 + 4 - 4x + y^2 + 4 - 4y = 13 = 0$$

$$x^2 + y^2 - 4x - 4y - 5 = 0$$



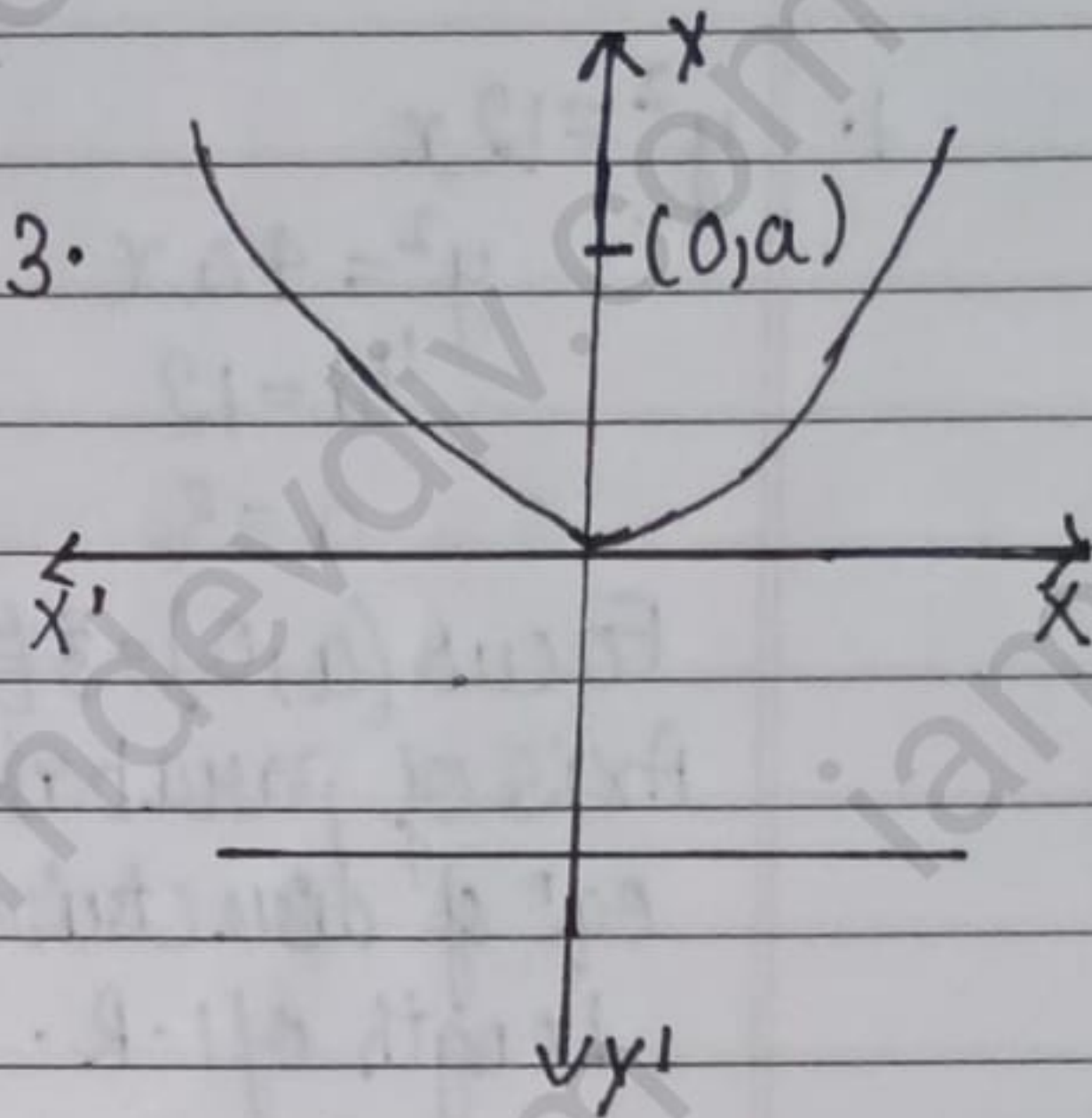
Parabola

1.)

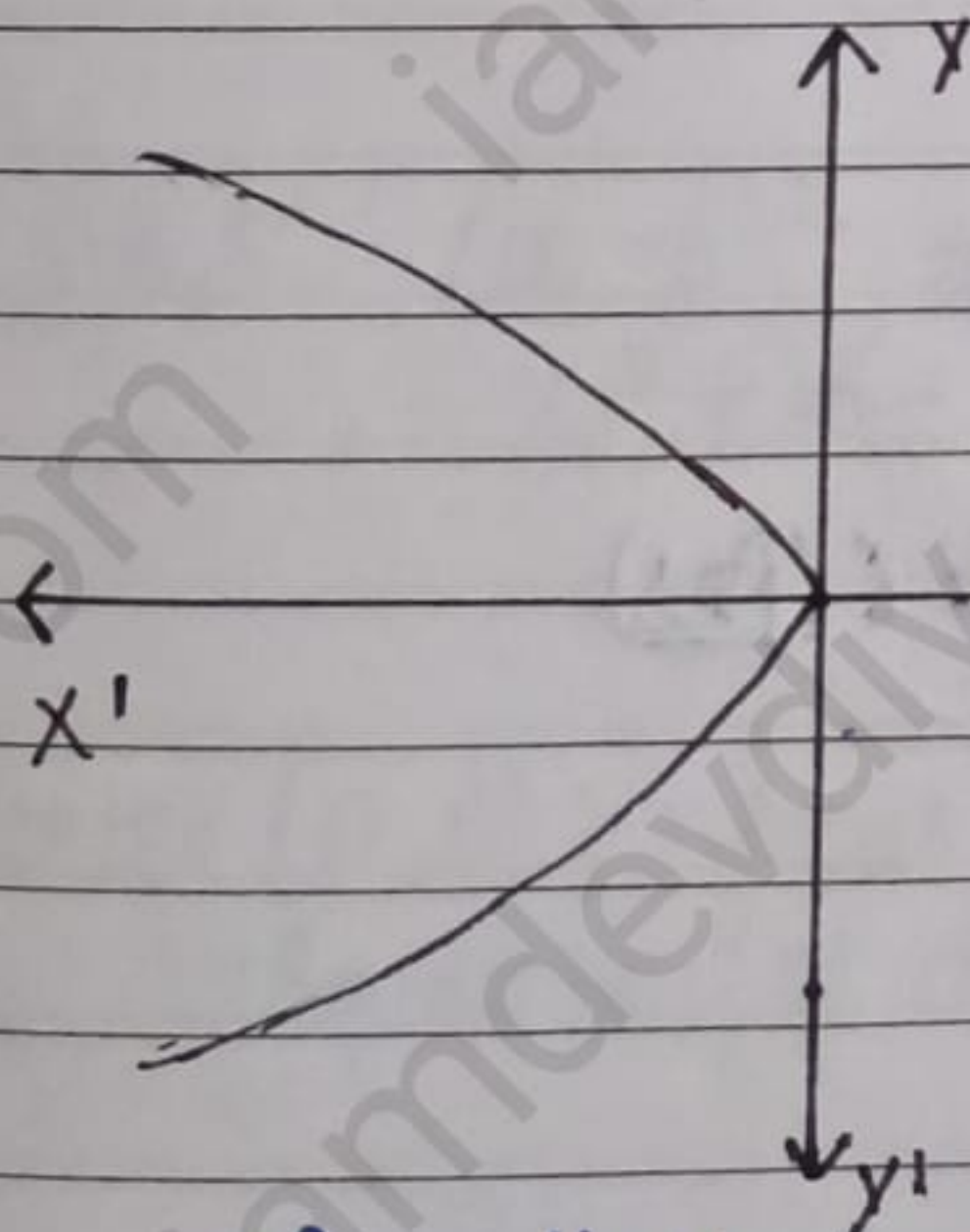


$y^2 = 4ax$
 Focus $F(a,0)$
 directrix $x + a = 0$
 Latus rectum $= 4a$
 axis $= x$ -axis

3.

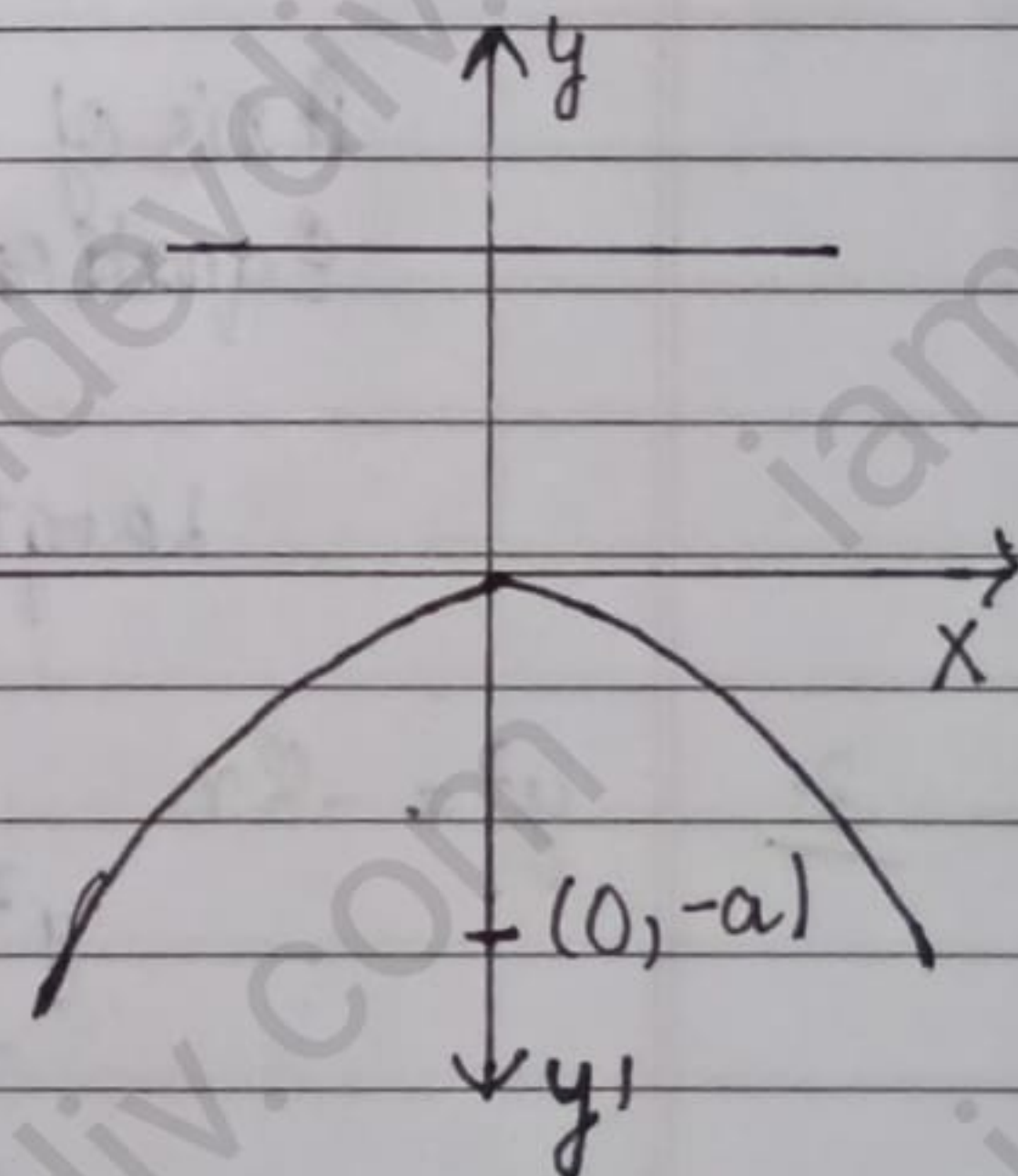


2.)



$y^2 = -4ax$
 $F = (-a,0)$
 directrix $x - a = 0$
 L.R. $= 4a$
 axis $= x$ -axis

4.



$x^2 = -4ay$
 $F = (0,-a)$
 directrix $y - a = 0$
 L.R. $= 4a$
 axis $= y$ -axis

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Exercise = 11.2

1. $y^2 = 12x$

$y^2 = 4ax$

$4a = 12$

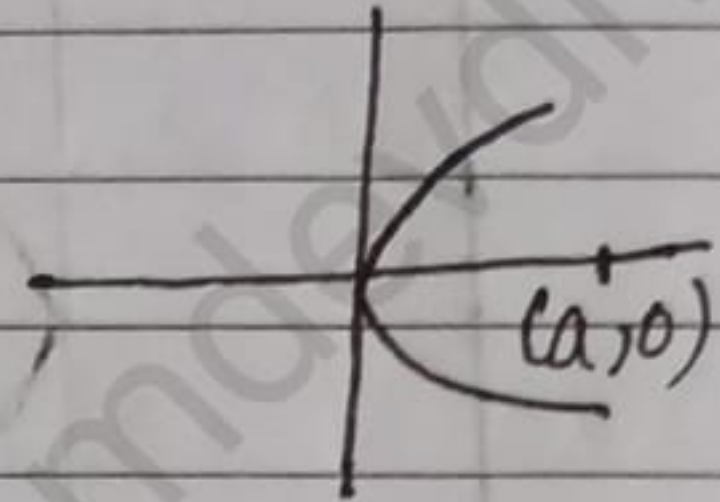
$a = 3$

Focus $(a, 0) = \cancel{3} (3, 0)$

Axis of parabola = x-axis

Eqⁿ of directrix = $x + 3 = 0$

Length of L.R. = $4a = 12$



2. $x^2 = 6y$

$x^2 = 4ay$

$4a = 6$

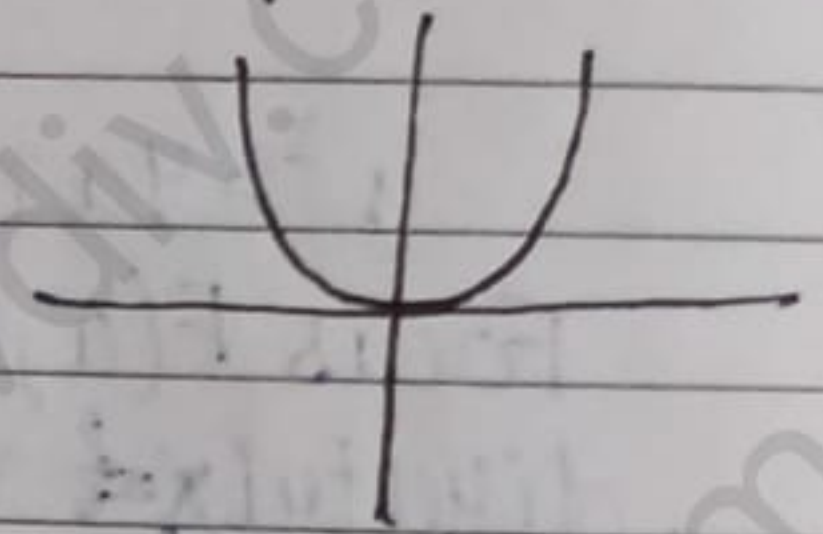
~~$a = 3$~~ $a = \frac{3}{2}$

Focus $(0, a) = (0, \frac{3}{2})$

Axis of parabola = y-axis

Equation of directrix = $y + \frac{3}{2} = 0$

Length of L.R. $4a = 4 \times \frac{3}{2} = 6$ (Ans)



3. $y^2 = -8x$

$y^2 = -4ax$

$-4a = -8$

$a = 2$

Focus $(-a, 0) = (-2, 0)$

Axis of parabola = x-axis

Eqⁿ of directrix = $x - 2 = 0$

L.R. = 8

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4. $x^2 = -16y$
 $x^2 = -4ay$
 $4a = 16$
 $a = 4$

Focus $(0, -a) = (0, -4)$
 Axis of parabola = y-axis
 Eqⁿ of directrix = $y - 4 = 0$
 Length of L.R. = $4a = 16$

7. Focus $(6, 0)$; directrix $x = 6$

$x = -6$
 $x + 6 = 0$
 P(x, y)
 PF = PM

$$\sqrt{(x-6)^2 + (y-0)^2} = \frac{x+6}{\sqrt{(1)^2}}$$

S.B.S. $(x-6)^2 + y^2 = (x+6)^2$
 $x^2 + 36 - 12x + y^2 = x^2 + 36 + 12x$
 $y^2 = 24x$

10. Vertex $(0, 0)$; focus $(-2, 0)$
 Equation of directrix $\Rightarrow x - 2 = 0$

P(x, y)
 PF = PM

$$\sqrt{(x+2)^2 + (y-0)^2} = \frac{x-2}{\sqrt{(1)^2}}$$

$$(x+2)^2 + y^2 = (x-2)^2$$

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$$x^2 + 4 + 4x + y^2 = x^2 + 4 - 4x$$

$$4x + y^2 = -4x$$

$$y^2 = -4x - 4x$$

$$y^2 = -8x$$

11. Vertex (0,0) passing through (2,3) and axis is along x-axis

Let the equation of parabola whose vertex (0,0) and axis is along x-axis

$$y^2 = 4ax$$

It passes through (2,3)

$$3^2 = 4a \times 2$$

$$9 = 8a$$

$$a = \frac{9}{8}$$

$$y^2 = 4ax$$

$$y^2 = 4 \times \frac{9}{8} x$$

$$2y^2 = 9x$$

5. $y^2 = 10x$

$$y^2 = 4ax$$

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

$$\text{Focus} = \left(\frac{5}{2}, 0 \right)$$

Axis of parabola = x-axis

Equation of directrix $\Rightarrow x + \frac{5}{2} = 0$, $2x + 5 = 0$

Latus Rectum $\Rightarrow 4a = 4 \times \frac{5}{2} = 10$

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5. $x^2 = -9y$

$$x^2 = -4ay$$

$$-4a = -9 \Rightarrow a = \frac{9}{4}$$

Focus = $(0, -\frac{9}{4})$

Axis of parabola = y-axis

Directrix = $y - \frac{9}{4} = 0$, $4y - 9 = 0$

Latus Rectum = $4a \Rightarrow 4 \times \frac{9}{4} = 9$

8. Focus $(0, -3)$; directrix $y = 3$

P (x, y)
 $PF = PM$

$$\sqrt{(x-0)^2 + (y+3)^2} = \frac{y-3}{\sqrt{(1)^2}}$$

$$x^2 + (y+3)^2 = (y-3)^2$$

$$x^2 + y^2 + 6y + 9 = y^2 - 6y + 9$$

$$x^2 = -12y$$

9. Vertex $(0, 0)$; focus $(3, 0)$

directrix $x = -3$

P (x, y)
 $PF = PM$

$$\sqrt{(x-3)^2 + (y-0)^2} = \frac{x+3}{\sqrt{(1)^2}}$$

$$(x-3)^2 + y^2 = (x+3)^2$$

$$x^2 - 6x + 9 + y^2 = x^2 + 6x + 9$$

$$y^2 = 12x$$

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12. Vertex $(0,0)$ passing through $(5,2)$ and symmetric with respect to y -axis

Let the equation be $x^2 = 4ay$

It passes through $(5,2)$

$$(5)^2 = 4a \times 2$$

$$25 = 8a$$

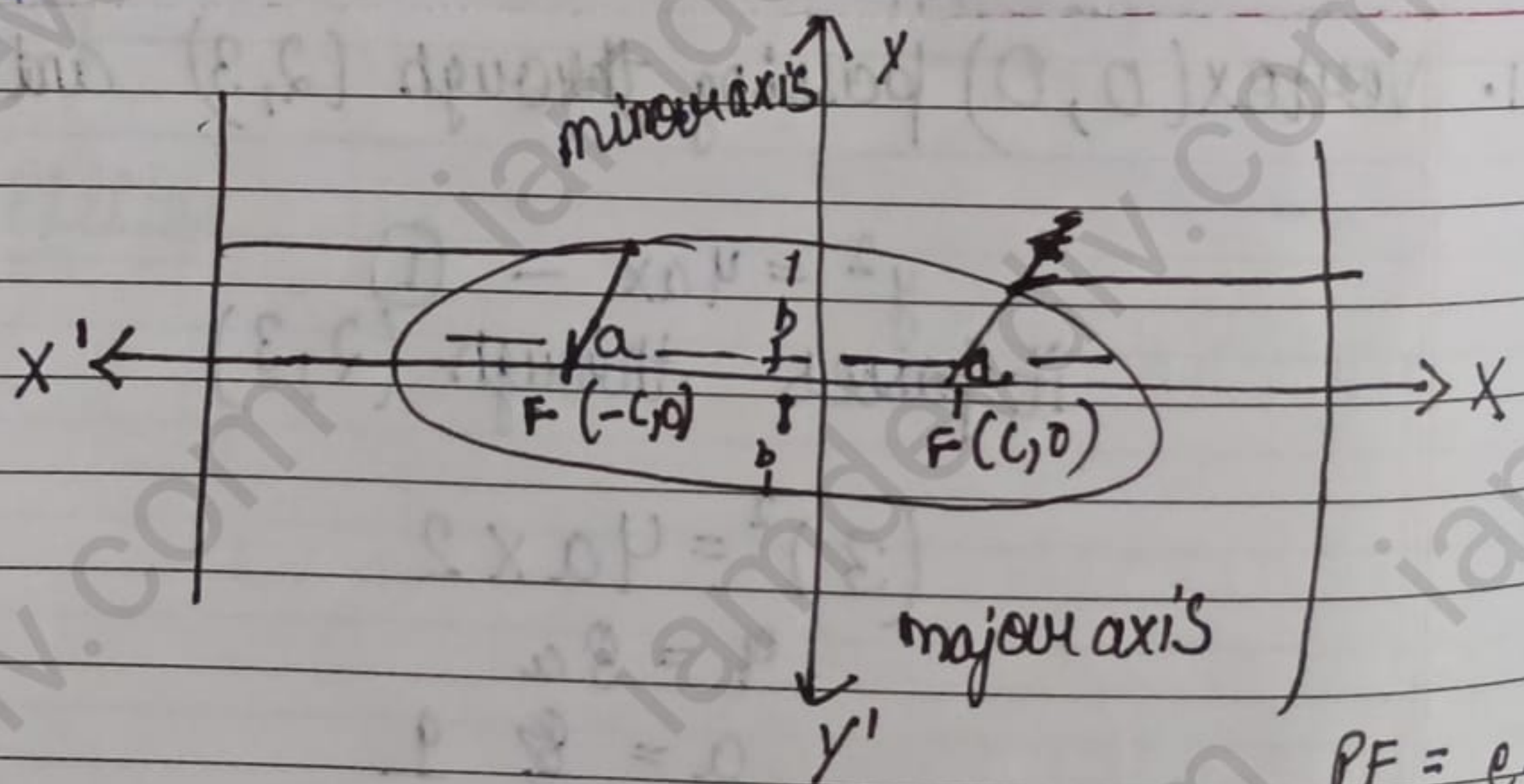
$$a = \frac{25}{8}$$

$$x^2 = 4ay$$

$$x^2 = 4 \times \frac{25}{8} y$$

$$x^2 = \frac{25}{2} y$$

$$= 2x^2 - 25y$$



$$PF = e PM$$

$$c = ae$$

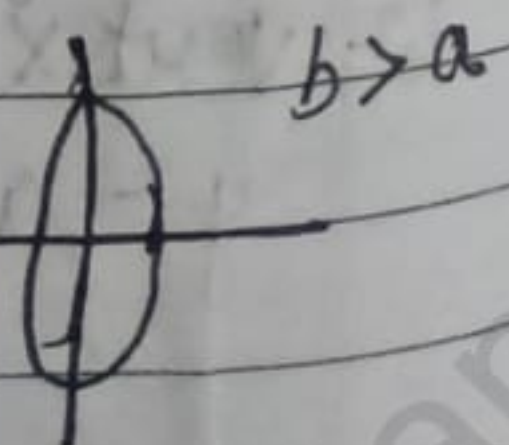
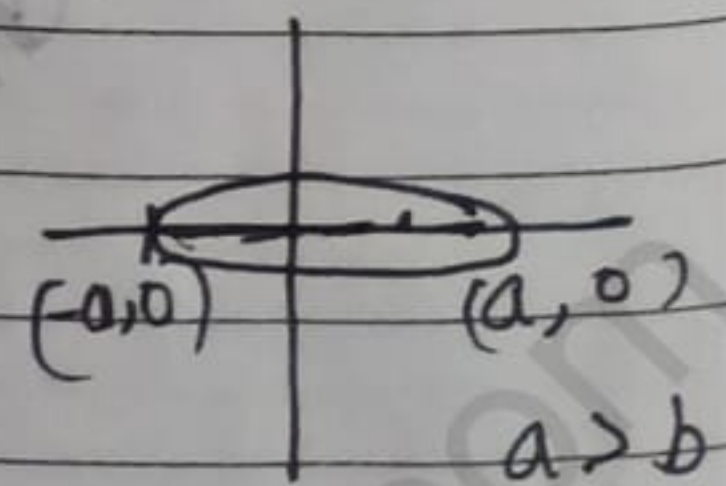
$$\frac{PF}{PM} = e < 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Length of major axis = $2a$

Length of minor axis = $2b$

$$c^2 = a^2 + b^2$$



Length of Latus rectum = $\frac{2b^2}{a}$

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Exercise = 11.3

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

$$\frac{x^2}{6^2} + \frac{y^2}{4^2} = 1$$

$$a=6 \quad b=4$$

$$c^2 = a^2 - b^2$$

$$c^2 = 6^2 - 4^2$$

$$c^2 = 36 - 16$$

$$c^2 = 20$$

$$c = 2\sqrt{5}$$

$$\text{Foci } (\pm c, 0)$$

$$\text{Foci } = (\pm 2\sqrt{5}, 0)$$

$$\text{Vertices } (\pm a, 0)$$

$$\text{Vertices } = (\pm 6, 0)$$

$$\text{Length of major axis} = 2a \\ = 2 \times 6 = 12$$

$$\text{Length of minor axis} = 2b \\ = 2 \times 4 = 8$$

$$e = \frac{c}{a} = \frac{2\sqrt{5}}{6} \quad e = \frac{\sqrt{5}}{3}$$

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times 4^2}{6} = \frac{16}{3} \quad (\text{Ans})$$

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$$8: 16x^2 + y^2 = 16$$

$$\frac{16x^2}{16} + \frac{y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\frac{x^2}{1^2} + \frac{y^2}{4^2} = 1$$

$$a=1$$

$$b=4$$

$$b > a \quad (\text{major axis} = y)$$

$$c^2 = b^2 - a^2$$

$$c^2 = 4^2 - 1^2$$

$$c = \sqrt{15}$$

$$\text{Foci} = (0, \pm c)$$

$$\text{Foci} = (0, \pm \sqrt{15})$$

$$\text{Vertices} = (0, \pm b)$$

$$\text{Vertices} = (0, \pm 4)$$

$$\text{Length of major axis} = 2b = 2 \times 4 = 8$$

$$\text{Length of minor axis} = 2a = 2 \times 1 = 2$$

$$\frac{c}{b} \Rightarrow e = \frac{\sqrt{15}}{4}$$

$$\text{Length of latus rectum} = \frac{2a^2}{b}$$

$$\frac{2 \times 1^2}{4} = \frac{1}{2} \quad (\text{Ans})$$

10. Vertices $(\pm 5, 0)$ foci $(\pm 4, 0)$

Vertices $= (\pm a, 0)$

$a = 5$

Foci $= (\pm c, 0)$

$c = 4$

$c^2 = a^2 - b^2$

$b^2 = a^2 - c^2$

$b^2 = 5^2 - 4^2$

$b^2 = 9$

$b = 3$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$

Eqⁿ of ellipse $= \frac{x^2}{25} + \frac{y^2}{9} = 1$

11. Vertices $(0, \pm 13)$ foci $(0, \pm 5)$

Foci $(0, \pm 5)$

$(0, \pm c)$

$c = 5$

Vertices $(0, \pm 13)$

Vertices $(0, \pm b)$

$b = 13$

$c^2 = b^2 - a^2$

$a^2 = b^2 - c^2$

$a^2 = 13^2 - 5^2$

$a^2 = 169 - 25$

$a = 12$

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$$\text{Eq}^n \text{ of ellipse} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$$

$$\boxed{\frac{x^2}{144} + \frac{y^2}{169} = 1}$$

13. Ends of major axis $(\pm 3, 0)$ ends of minor axis $(0, \pm 2)$

$$\text{Major axis} = (\pm a, 0)$$

$$\boxed{a = 3}$$

Minor axis $(0, \pm b)$

$$\boxed{b = 2}$$

$$\text{Eq}^n \text{ of ellipse} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

15. Length of major axis 26, foci $(\pm 5, 0)$

$$\text{Major axis} = 2a$$

$$2a = 26$$

$$\boxed{a = 13}$$

foci $(\pm c, 0)$

$$c = 5$$

$$c^2 = a^2 - b^2$$

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$$b^2 = a^2 - c^2$$

$$b^2 = 13^2 - 5^2$$

$$b^2 = 169 - 25$$

$$b = 12$$

$$\text{Eq}^n \text{ of ellipse} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$$

$$\frac{x^2}{169} + \frac{y^2}{144} = 1$$

17 Foci ($\pm 3, 0$)

$$a = 4$$

Foci ($\pm 3, 0$)

Foci ($\pm c, 0$)

$$c = 3$$

$$c^2 = a^2 - b^2$$

$$b^2 = a^2 - c^2$$

$$b^2 = 4^2 - 3^2$$

$$b^2 = 16 - 9$$

$$b = \sqrt{7}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{\sqrt{7}^2} = 1$$

$$\text{Eq}^n \text{ of ellipse} = \frac{x^2}{16} + \frac{y^2}{7} = 1$$

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19. Centre at (0,0) major axis on the y-axis and passes through the points (3,2) and (1,6)

Let eqⁿ of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ — (1)

(1) passes through (3,2)

$$\frac{3^2}{a^2} + \frac{2^2}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$
 — (2)

(2) passes through (1,6)

$$\frac{1^2}{a^2} + \frac{6^2}{b^2} = 1$$

$$\frac{1}{a^2} + \frac{36}{b^2} = 1$$
 — (3)

$$(2) \times 9 - (3)$$

$$\frac{81}{a^2} + \frac{36}{b^2} = 9$$

$$\frac{1}{a^2} + \frac{36}{b^2} = 1$$

$$\frac{1080}{a^2} = 8$$

$$\boxed{a^2 = 10}$$

$$\frac{1}{10} + \frac{36}{b^2} = 1$$

$$\frac{36}{b^2} = 1 - \frac{1}{10} \Rightarrow \frac{36}{b^2} = \frac{9}{10} \Rightarrow \boxed{b^2 = 40}$$

$$\boxed{\text{eq}^n \text{ of Ellipse} = \frac{x^2}{10} + \frac{y^2}{40} = 1}$$

20. Major axis on the x-axis and passes through the points (4, 3) and (6, 2)

Let eqⁿ of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (1)}$$

① passes through (4, 3)

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \quad \text{--- (2)}$$

① passes through (6, 2)

$$\frac{36}{a^2} + \frac{4}{b^2} = 1 \quad \text{--- (3)}$$

From eqⁿ (2)

$$\frac{16}{a^2} = \frac{b^2 - 9}{b^2}$$

$$a^2 = \frac{16b^2}{b^2 - 9}$$

Put it in eqⁿ (3)

$$\frac{9 \cdot 36}{4 \cdot 16 \cdot b^2} (b^2 - 9) + \frac{4}{b^2} = 1$$

$$9b^2 - 81 + 16 = 4b^2$$

$$5b^2 = 65$$

$$\boxed{b^2 = 13}$$

$$a^2 = \frac{16 \times 13}{13 - 9}$$

$$13 - 9$$

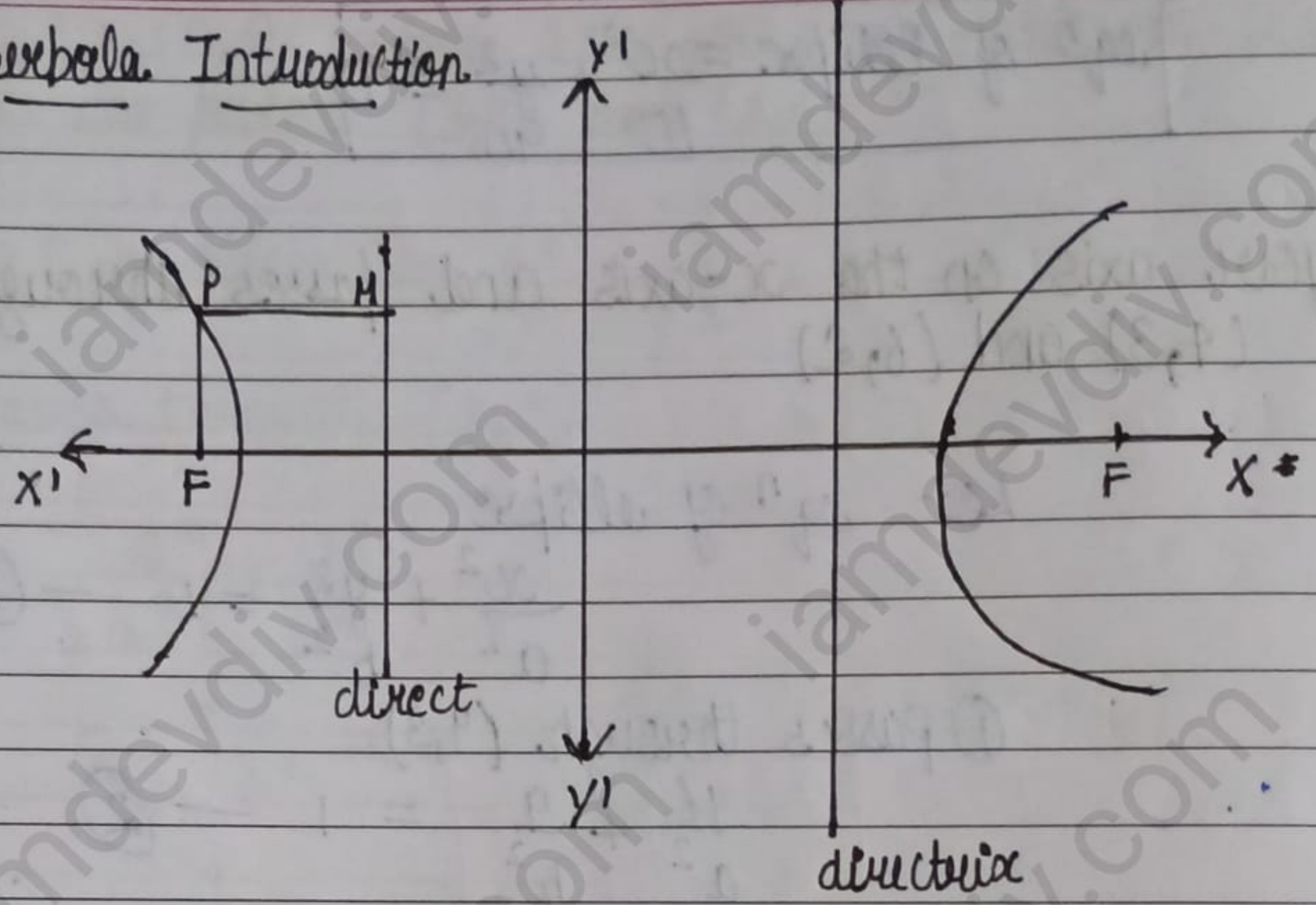
$$a^2 = \frac{208}{4} = 52$$

Eqⁿ of ellipse

$$\boxed{\frac{x^2}{52} + \frac{y^2}{13} = 1}$$

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Hyperbola Introduction



Length of conjugate axis = $2a$
 Length of transverse axis = $2b$
 distance b/w two foci = $2ae$

$$\frac{PF}{PM} = e$$

$$e > 1$$

Foci $(\pm c, 0)$
 directrix $x = \pm ae$
 $c^2 = a^2 + b^2$

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Exercise = 11.4

$$1. \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$a = 4, \quad b = 3$$

$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2$$

$$c^2 = 16 + 9$$

$$c = \sqrt{25} = 5$$

$$\text{Foci } (\pm c, 0)$$

$$(\pm 5, 0)$$

$$\text{Vertices } (\pm a, 0) = (\pm 4, 0)$$

$$e = \frac{c}{a} = \frac{5}{4}$$

$$\text{Length of L.R.} = \frac{2b^2}{a}$$

$$= \frac{2 \times 3^2}{4} = \frac{9}{2} \text{ (Ans)}$$

$$2. \frac{y^2}{9} - \frac{x^2}{27} = 1$$

$$\frac{y^2}{3^2} - \frac{x^2}{(3\sqrt{3})^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a = 3, \quad b = 3\sqrt{3}$$

$$c^2 = a^2 + b^2$$

$$c = \sqrt{9 + 27} = 6$$

$$\text{Foci } (0, \pm c) = (0, \pm 6)$$

$$\text{Vertices } (0, \pm a) = (0, \pm 3)$$

$$e = \frac{c}{a} = \frac{6}{3} = 2$$

$$\text{L.R.} = \frac{2b^2}{a}$$

$$= \frac{2 \times 27 \times 9}{3}$$

$$= 18 \text{ (Ans)}$$

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3. $9y^2 - 4x^2 = 36$

$\frac{9y^2}{36} - \frac{4x^2}{36} = 1$

$\frac{y^2}{4} - \frac{x^2}{9} = 1$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$a=2 \quad b=3$

$c = \sqrt{4+9}$

$c = \sqrt{13}$

Foci $(0, \pm c)$

$(0, \pm \sqrt{13})$

Vertices $(0, \pm a)$

$(0, \pm 2)$

$\frac{c}{a} = \frac{\sqrt{13}}{2}$

L.R. = 2×9

$= 2$

$= 9 \text{ (Ans)}$

4. $16x^2 - 9y^2 = 576$

$\frac{16x^2}{576} - \frac{9y^2}{576} = 1$

$a=3 \quad b=4$

$c = \sqrt{16+9}$

$c = \sqrt{25}$
 $= 5$

Foci $(0, \pm c)$

$(0, \pm 5)$

$\frac{c}{a} = \frac{5}{3}$

Vertices $(0, \pm a)$

$(0, \pm 3)$

L.R. = 2×16

$= 3$

$\frac{c}{a} = \frac{5}{3}$

Ques - In each - - - - - conditions.

8. Vertices $(0, \pm 5)$, foci $(0, \pm 8)$
 $0, \pm b$ $0, \pm c$

$$b = 5, c = 8$$

$$c^2 = a^2 + b^2$$

$$8^2 = a^2 + 5^2$$

$$64 - 25 = a^2$$

$$a^2 = 39$$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$\frac{y^2}{25} - \frac{x^2}{39} = 1$$

10. Foci $(\pm 5, 0)$ $(\pm c, 0)$ $c = 5$
 Transverse axis = 8

$$2a = 8$$

$$a = 4$$

$$c^2 = a^2 + b^2$$

$$5^2 = 4^2 + b^2$$

$$b^2 = 25 - 16$$

$$b^2 = 9$$

eqⁿ of hyperbola

$$\frac{x^2}{4^2} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

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~~H. Foci(0, ±13)~~

13. Foci(±4, 0) (±c, 0) = 4

Latus Rectum = 12

~~$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$~~

$\frac{x^2}{4^2} - \frac{y^2}{12} = 1$

Latus Rectum = $\frac{2b^2}{a}$

~~12 = $\frac{2b^2}{a} = 12$~~

$b^2 = 6a$

$c^2 = a^2 + b^2$

$c^2 = a^2 + 6a$

$= a^2 + 6a - 16 = 0$

$= a^2 + 8a - 2a - 16 = 0$

$a(a+8) - 2(a+8) = 0$

$(a-2)(a+8) = 0$

$a = 2 \quad a = -8$ (invalid)

$a^2 = 4 \quad b^2 = 12$

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

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15. Foci $(0, \pm \sqrt{10})$, passing through $(2, 3)$

$$(0, \pm c)$$

$$c = \sqrt{10}$$

passing through $(2, 3)$

$$c^2 = a^2 + b^2$$

$$(\sqrt{10})^2 =$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad (1)$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{9}{a^2} - \frac{4}{b^2} = 1$$

$$\frac{4x^2}{a^2} - \frac{2y^2}{b^2} = 1$$

$$2y^2 = 4x^2 - 1$$

$$9b^2 - 4a^2 = a^2b^2 \quad (2)$$

$$c^2 = a^2 + b^2$$

$$\frac{4}{5} - \frac{2}{5} = 1$$

$$10 = a^2 + b^2$$

$$a^2 = 10 - b^2$$

Put value of a^2 in eqⁿ (2)

$$9b^2 - 4(10 - b^2) = 10b^2 - b^4$$

$$b^4 + 3b^2 - 40 = 0$$

$$b^4 + 8b^2 - 5b^2 - 40 = 0$$

$$b^2(b^2 + 8) - 5(b^2 + 8) = 0$$

$$(b^2 + 8) \cdot (b^2 - 5) = 0$$

$$b^2 = 5$$

$$a^2 = 10 - 5$$

$$a^2 = 5$$

Put these value in eqⁿ (1)

$$\frac{y^2}{5} - \frac{x^2}{5} = 1$$

$$\boxed{y^2 - x^2 = 5}$$

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Miscellaneous Exercise

1.

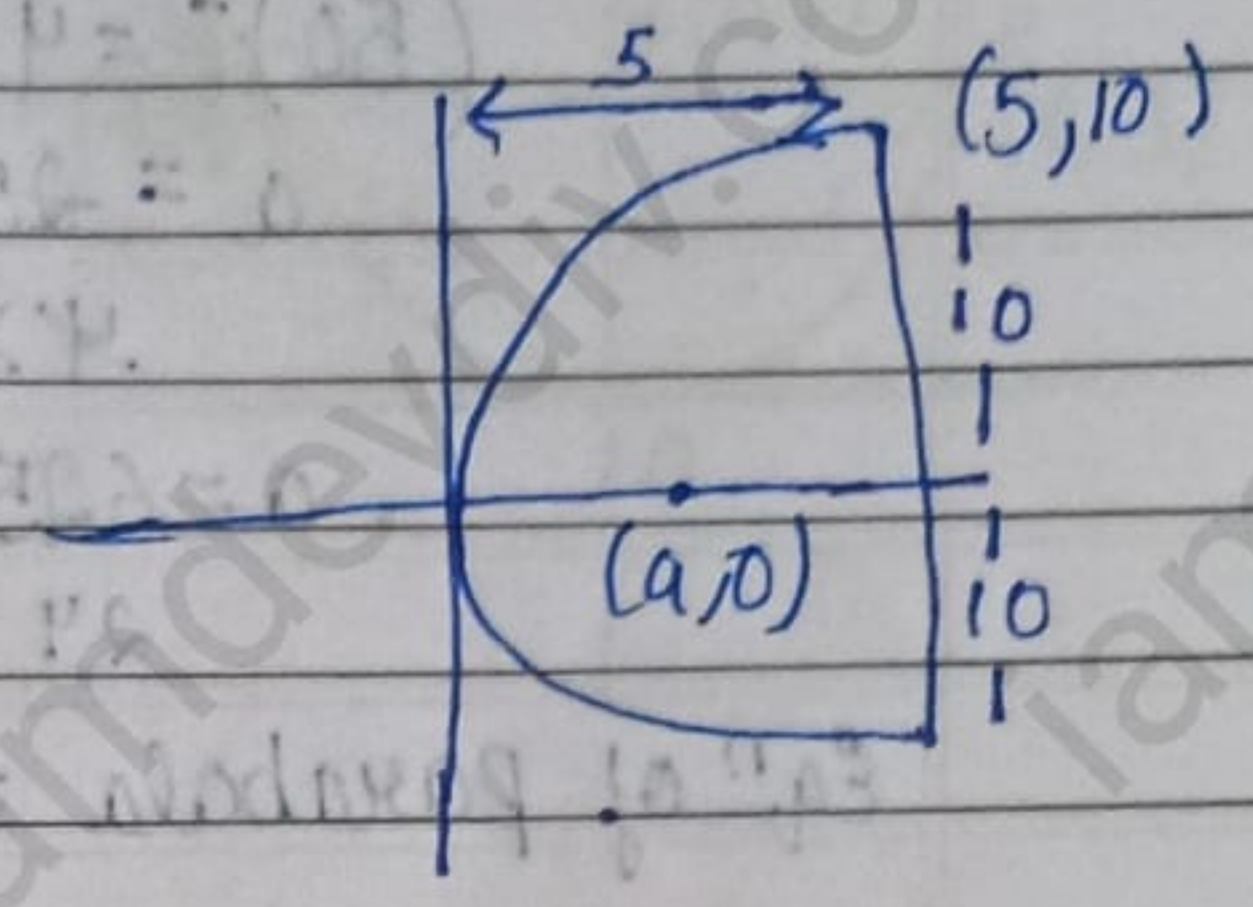
$$y^2 = 4ax$$

$$(10)^2 = 4a \times 5$$

$$100 = 20a$$

$$\boxed{a=5}$$

Focus (5, 0) (Ans)



2.

$$x^2 = 4ay$$

$$\left(\frac{5}{2}\right)^2 = 4a \times 10^2$$

$$\frac{5}{32} = a$$

$$x^2 = 4 \times \frac{5}{32} \times y$$

$$x^2 = \frac{5}{8} y$$

y = 2 then

$$x^2 = \frac{5}{8} \times 2$$

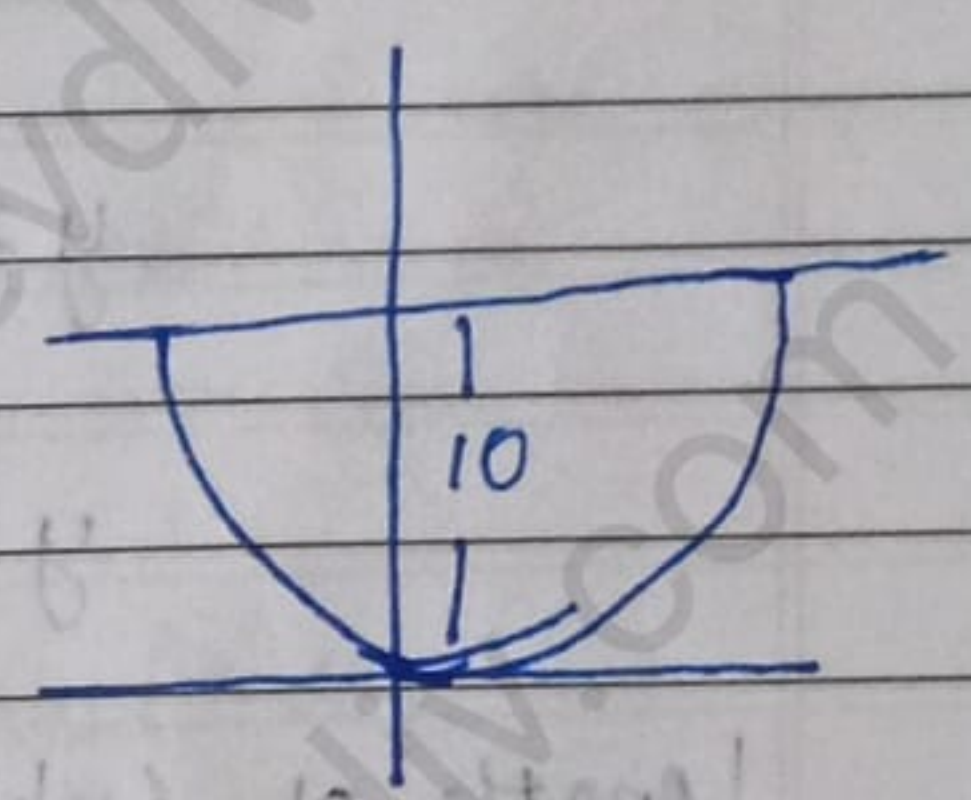
$$x^2 = \frac{5}{4}$$

$$x = \frac{\sqrt{5}}{2}$$

$$AB = 2x$$

$$= 2 \times \frac{\sqrt{5}}{2}$$

$$= 2.23$$



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3.

the eqⁿ of parabola is

$$x^2 = 4ay$$

it passes through (50, 24)

$$(50)^2 = 4a \times 24$$

$$a = \frac{2500}{625}$$

$$4 \times 24$$

$$a = 625$$

$$24$$

Eqⁿ of parabola $\Rightarrow x^2 = \frac{4 \times 625}{24} y$

$$x^2 = \frac{625}{6} y$$

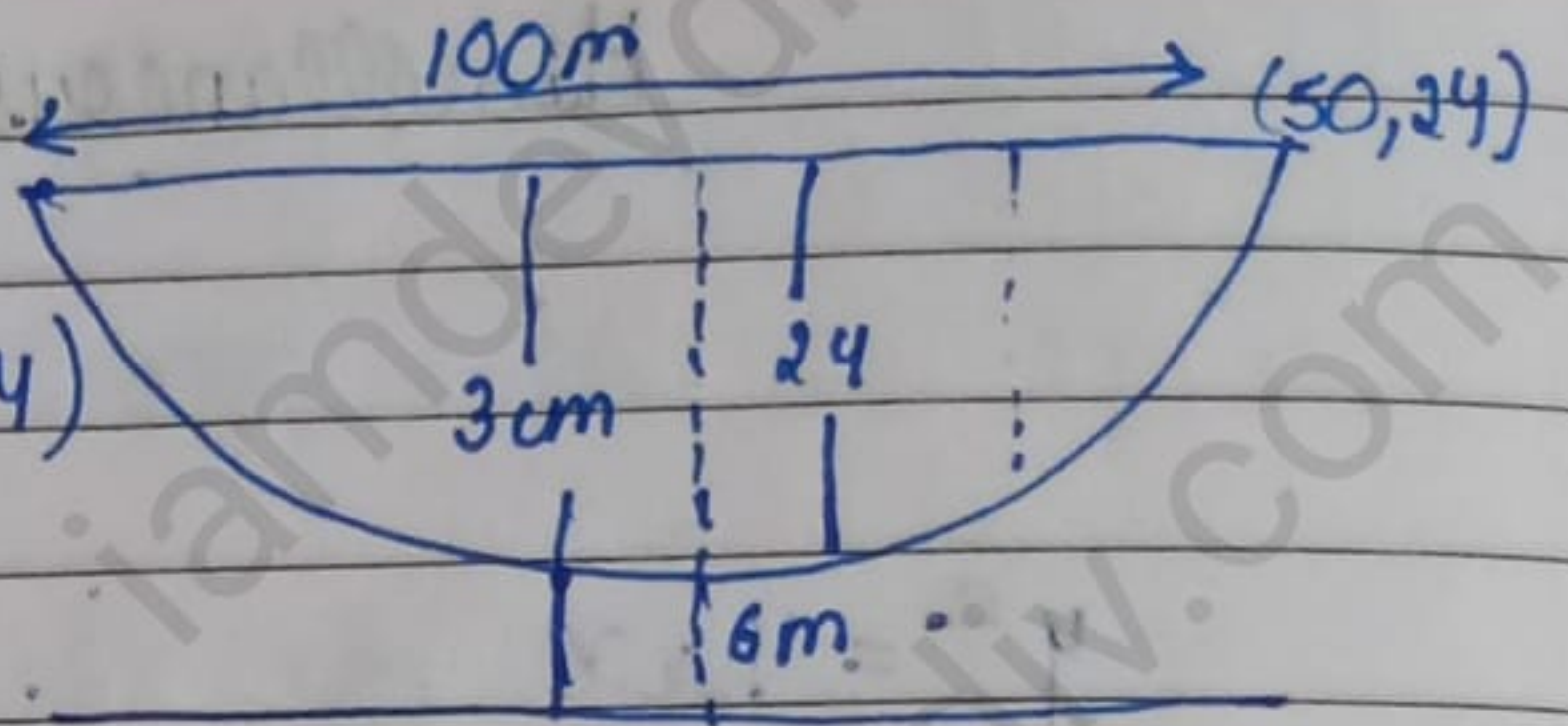
When $x = 18m$

$$(18)^2 = \frac{625}{6} y$$

$$y = \frac{18 \times 18 \times 6}{625}$$

$$y = \frac{1944}{625} = 3.11 \text{ Approx}$$

Length of supporting wire = $3.11 + 6m = 9.11m$ (Ans)



5.

$$AP = 3cm$$

$$PB = 9cm$$

In ΔPBA

$$\cos \theta = \frac{x}{9}$$

$$\angle BPA = \angle PAR = \theta$$

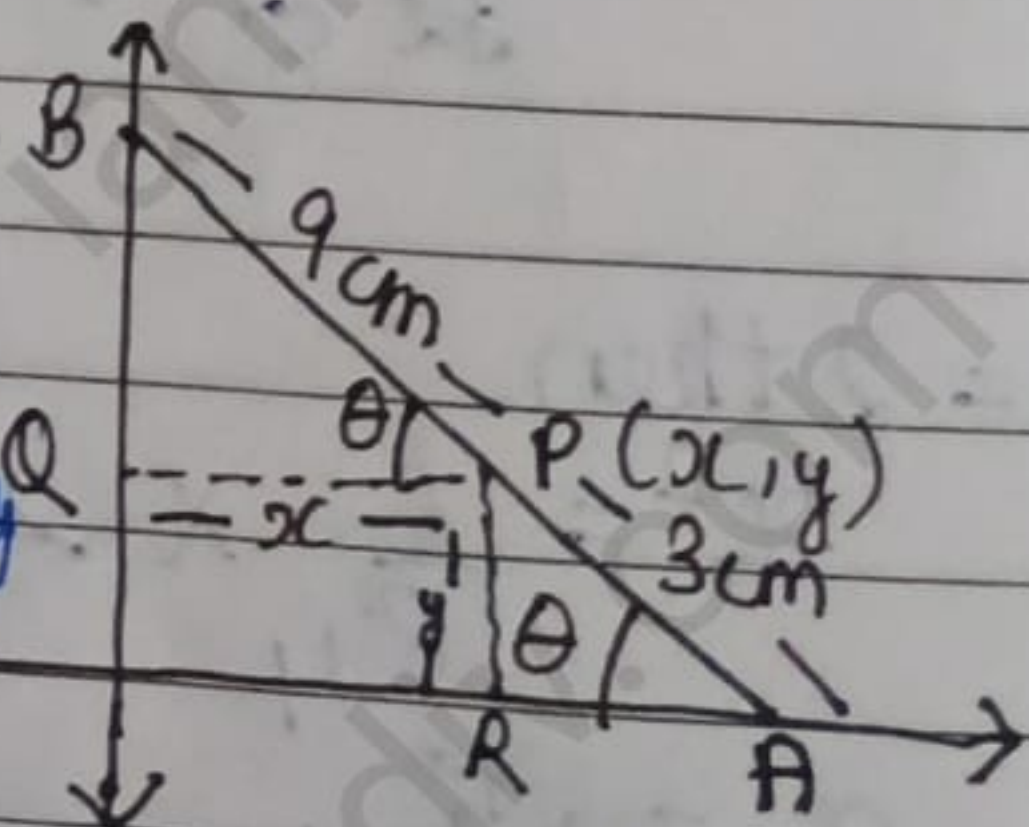
(Corresponding angles)

In ΔPRA ,

$$\sin \theta = \frac{y}{3}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

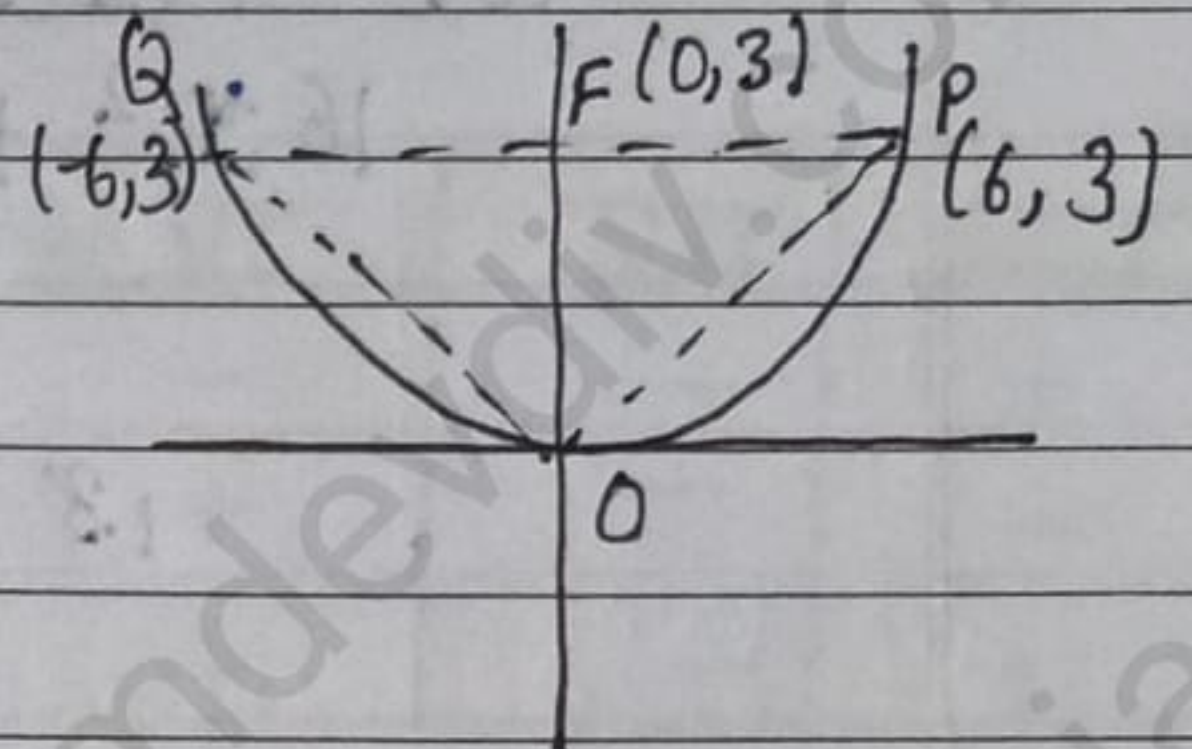
$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{9}\right)^2 = 1$$



$$\frac{y^2}{9} + \frac{x^2}{81} = 1 \Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$$

6.

$$\begin{aligned} x^2 &= 12y \\ x^2 &= 4ay \\ 4a &= 12 \\ a &= 3 \end{aligned}$$



$$\begin{aligned} y &= 3 \\ \textcircled{1} \Rightarrow x^2 &= 12 \times 3 = 36 \\ x &= \pm 6 \end{aligned}$$

$$\begin{aligned} O(0,0) \quad P(6,3) \quad Q(-6,3) \\ PQ = 6 + 6 = 12 \text{ cm} \quad OF = 3 \text{ cm} \end{aligned}$$

$$\text{Area } \Delta OPQ = \frac{1}{2} \times B \times H$$

$$= \frac{1}{2} \times PQ \times OF$$

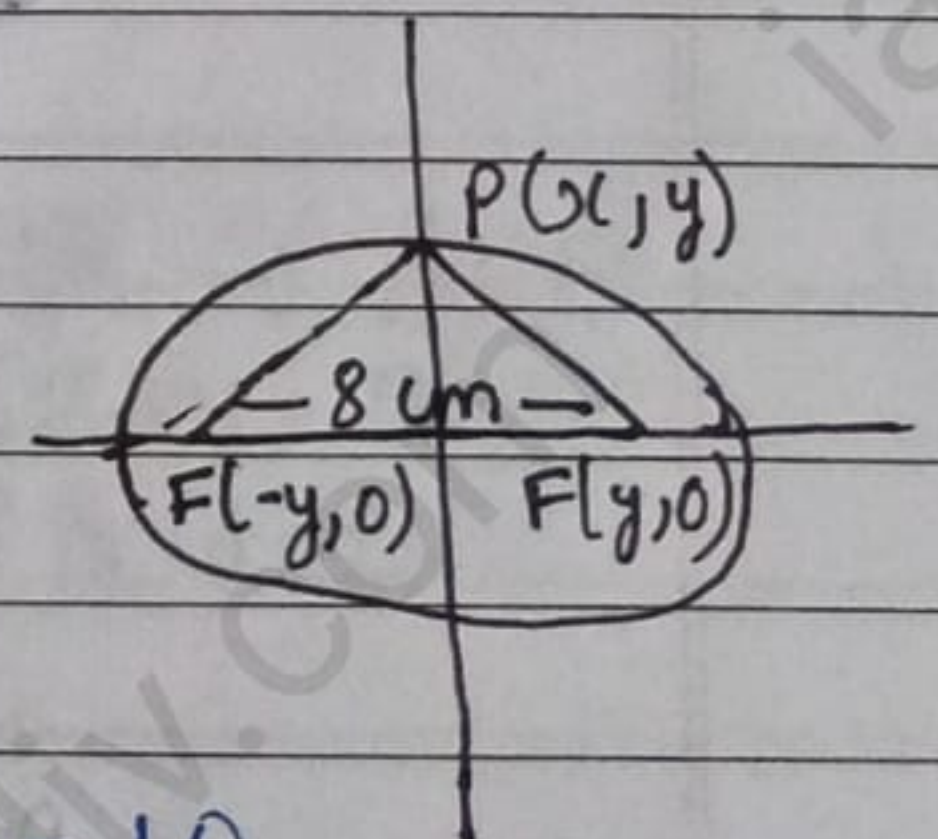
$$= \frac{1}{2} \times 12 \times 3 = 18 \text{ unit sq.}$$

7.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$FF' = 2c = 8 = c = 4$$

$$PF + PF' = 81$$



$$\sqrt{(x+4)^2 + (y-0)^2} + \sqrt{(x-4)^2 + (y-0)^2} = 10$$

$$\sqrt{(x+4)^2 + y^2} + \sqrt{(x-4)^2 + y^2} = 10$$

$$\sqrt{(x+4)^2 + y^2} = 10 - \sqrt{(x-4)^2 + y^2}$$

$$(x+4)^2 + y^2 = 100 + (x-4)^2 + y^2 - 20\sqrt{(x-4)^2 + y^2}$$

$$x^2 + 16 + 8x = 100 + x^2 + 16 - 8x - 20\sqrt{(x-4)^2 + y^2}$$

$$16x - 100 = -20\sqrt{(x-4)^2 + y^2}$$

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$$4(4x-25) = -20\sqrt{(x-4)^2 + y^2}$$

Squaring both sides

$$16x^2 + 625 - 200x = 25x[(x-4)^2 + y^2]$$

$$16x^2 + 625 - 200x = 25x^2 + 400x - 200x + 25y^2$$

$$9x^2 + 25y^2 + 400 - 625 = 0$$

$$9x^2 + 25y^2 - 225 = 0$$

$$18 \frac{9x^2}{225} + \frac{25y^2}{225} = 1$$

$$\frac{75}{25} \frac{x^2}{25} + \frac{45}{9} \frac{y^2}{9} = 1$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$

B.

when $x = k$

$$y^2 = 4ak$$

$$y = \pm \sqrt{4ak}$$

$$y = \pm 2\sqrt{ak}$$

$$O(0,0) \quad A(k, 2\sqrt{ak}) \quad B(k, -2\sqrt{ak})$$

$$OA = AB$$

$$\sqrt{k^2 + 4ak} = 2\sqrt{ak} + 2\sqrt{ak}$$

$$\sqrt{k^2 + 4ak} = 4\sqrt{ak}$$

S.B.S.

$$k^2 + 4ak = 16ak$$

$$k^2 = 12ak$$

$$\boxed{k = 12a}$$

$$AB = 4\sqrt{ak}$$

$$AB = 4\sqrt{a \cdot 12a} = 4a \times 2\sqrt{3} = 8\sqrt{3}a$$

Side of Equilateral triangle is $8\sqrt{3}a$.

